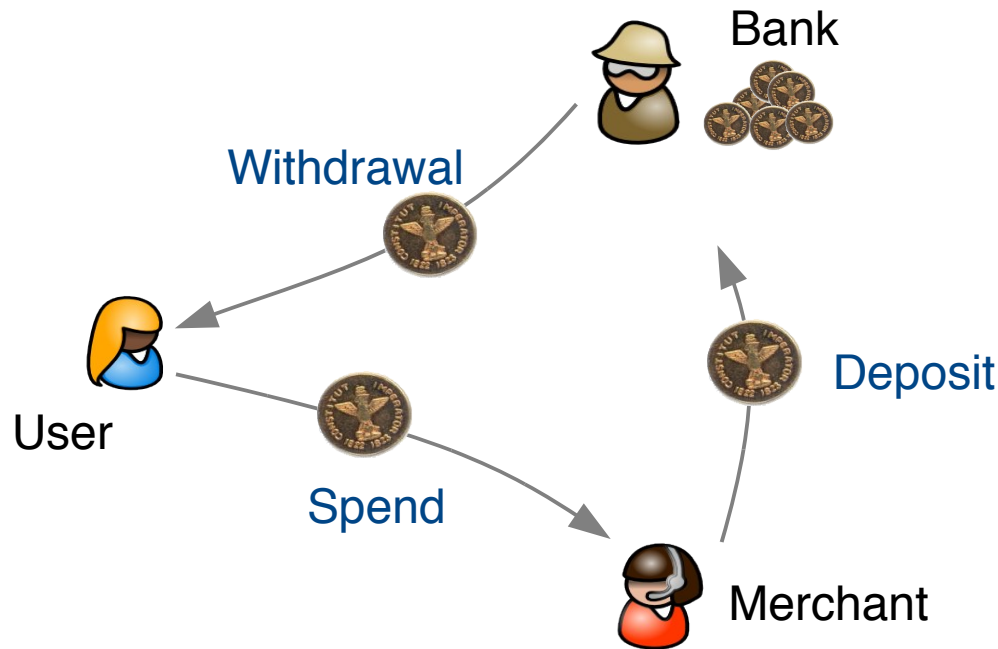


# Cryptographic e-Cash

Jan Camenisch

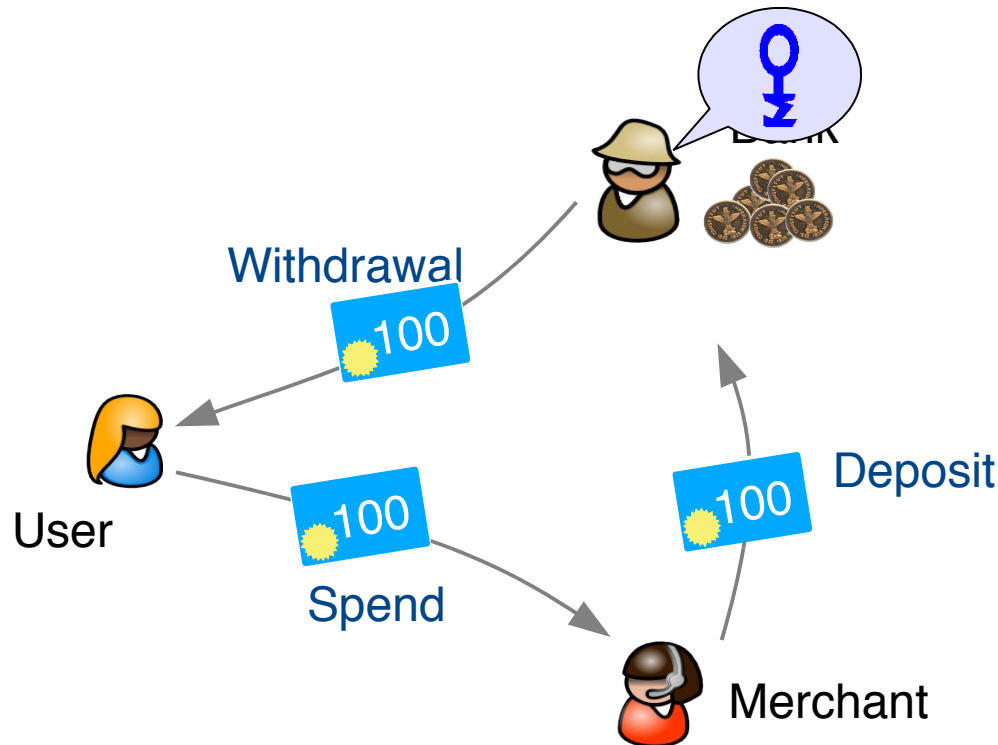
IBM Research – Zurich  
@JanCamenisch  
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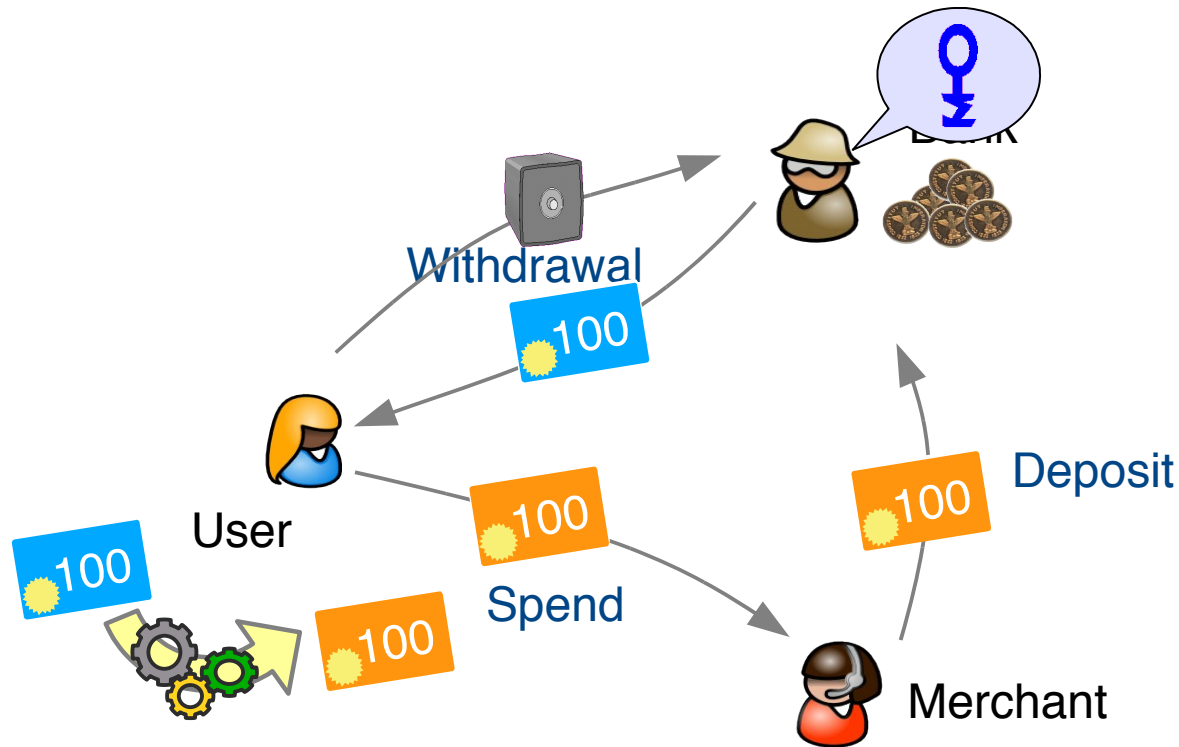


## Requirements

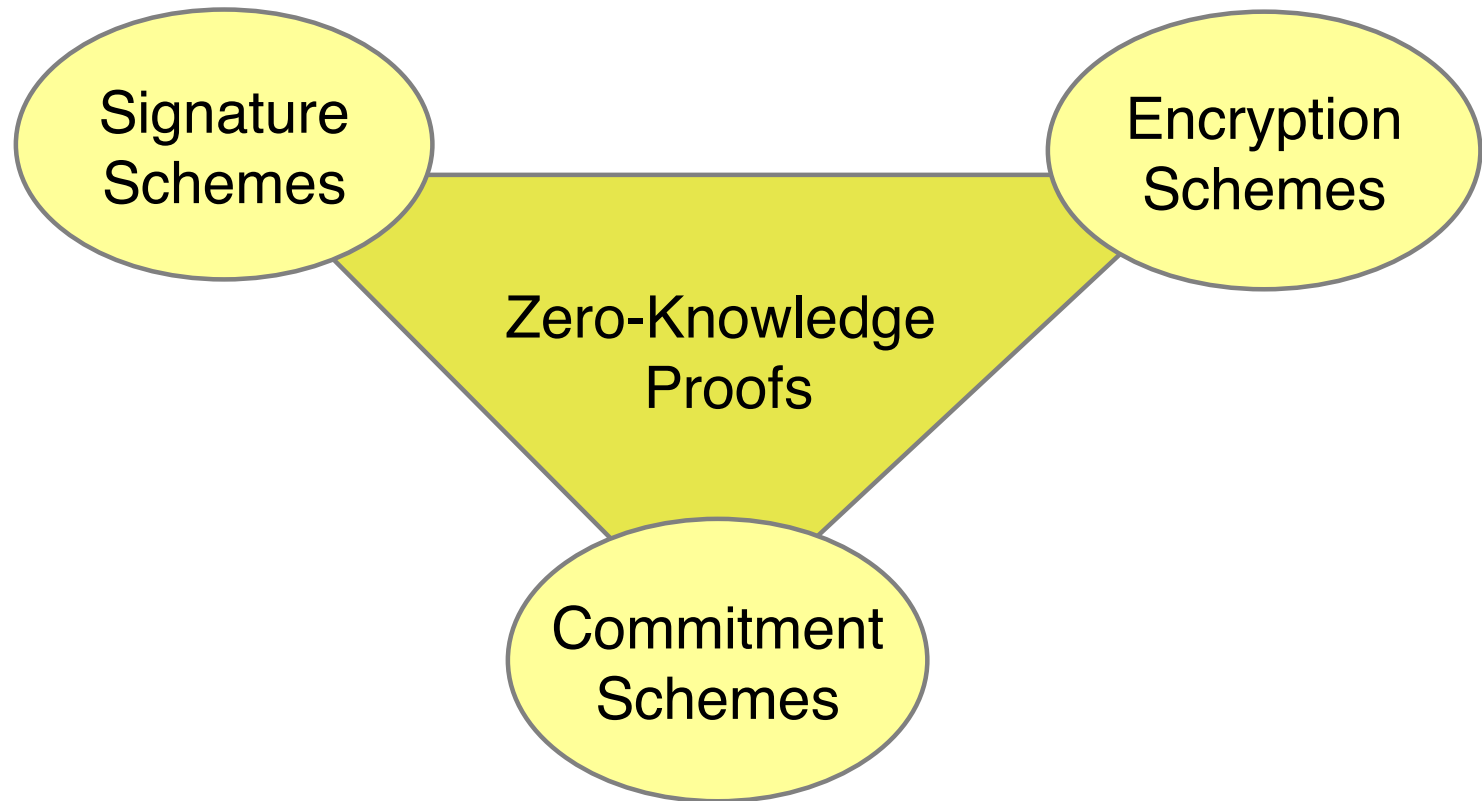
- **Anonymity:** Withdrawal and Deposit must be unlinkable
- **No Double Spending:** Coin is bit-strings, can be spend twice



- Sign notes with digital signature scheme
  - Note = (serial number #, value)
  - Secure because
    - signature scheme can not be forged
    - bank will accept some serial number only once → on-line e-cash
  - *Not* anonymous because (cf. paper solution)
    - bit-string of signature is unique
    - serial number is unique



- Use (more) cryptography
  - Hide serial number from bank when issuing
    - e.g., sign commitment of serial number
  - Reveal serial number and proof
    - knowledge of signature on
    - commitment to serial number
  - Anonymous because of commitments scheme and zero-knowledge proof



..... challenge is to do all this efficiently!

A photograph of a wooden crate, possibly a shipping container, resting on a rough, rocky, and uneven ground. The crate is made of light-colored wood and has several horizontal slats. The text "mathematical setting" is overlaid in a bold, blue, sans-serif font on the left side of the image, partially overlapping the crate.

**mathematical setting**

A set  $G$  with operation  $\square$  is called a **group** if:

– closure

for all  $a, b$ , in  $G \rightarrow a \square b$  in  $G$

– commutativity

for all  $a, b$ , in  $G \rightarrow a \square b = b \square a$

– associativity

for all  $a, b, c$ , in  $G \rightarrow (a \square b) \square c = a \square (b \square c)$

– identity

there exist some  $e$  in  $G$ , s.t. for all  $a$ :  $a \square e = a$

– invertibility

for all  $a$  in  $G$ , there exist  $a^{-1}$  in  $G$ :  $a \square a^{-1} = e$

▪ *Example:*

integers under addition  $(\mathbb{Z}, +) = \{\dots, -2, -1, 0, 1, 2, \dots\}$  or  $(\mathbb{Z}_n, +) = \{0, 1, 2, \dots, n-1\}$

identity:  $e = 0$

inverse:  $a^{-1} = -a$

- **exponentiation** = repeated application of  $\cdot$  , e.g.,  $a^3 = a \cdot a \cdot a$
- a group is **cyclic** if every element is power of some fixed element:
  - i.e., for each  $a$  in  $G$ , there is unique  $i$  such that  $g^i = a$
  - $g$  = generator of the group
  - define  $g^0 = 1$  = identity element
$$G = \langle g \rangle = \{1=g^0, g^1, g^2, \dots, g^{q-1}\}$$
  - $q = |G|$  = order of group
  - if  $q$  is a prime number then  $G$  is cyclic
  - $\rightarrow$  computation in exponents can be done **modulo**  $q$ :

$$g^i = g^{i \bmod q}$$

- computing with exponents:

$$g^{i+j} = g^i \cdot g^j$$

$$g^{i-j} = g^i / g^j = g^i \cdot (g^j)^{-1}$$

$$g^{ij} = (g^i)^j$$

$$g^{-i} = (g^{-1})^i = (g^i)^{-1}$$



given  $g$  and  $x$  it is **easy** to compute  $g^x, g^{1/x}, \dots$

given  $g^x$  and  $g^y$  it is **easy** to compute  $g^x g^y = g^{x+y}$

## Discrete Log Assumption

given  $g^x$  it is hard to *compute*  $x$

## Diffie-Hellman Assumption

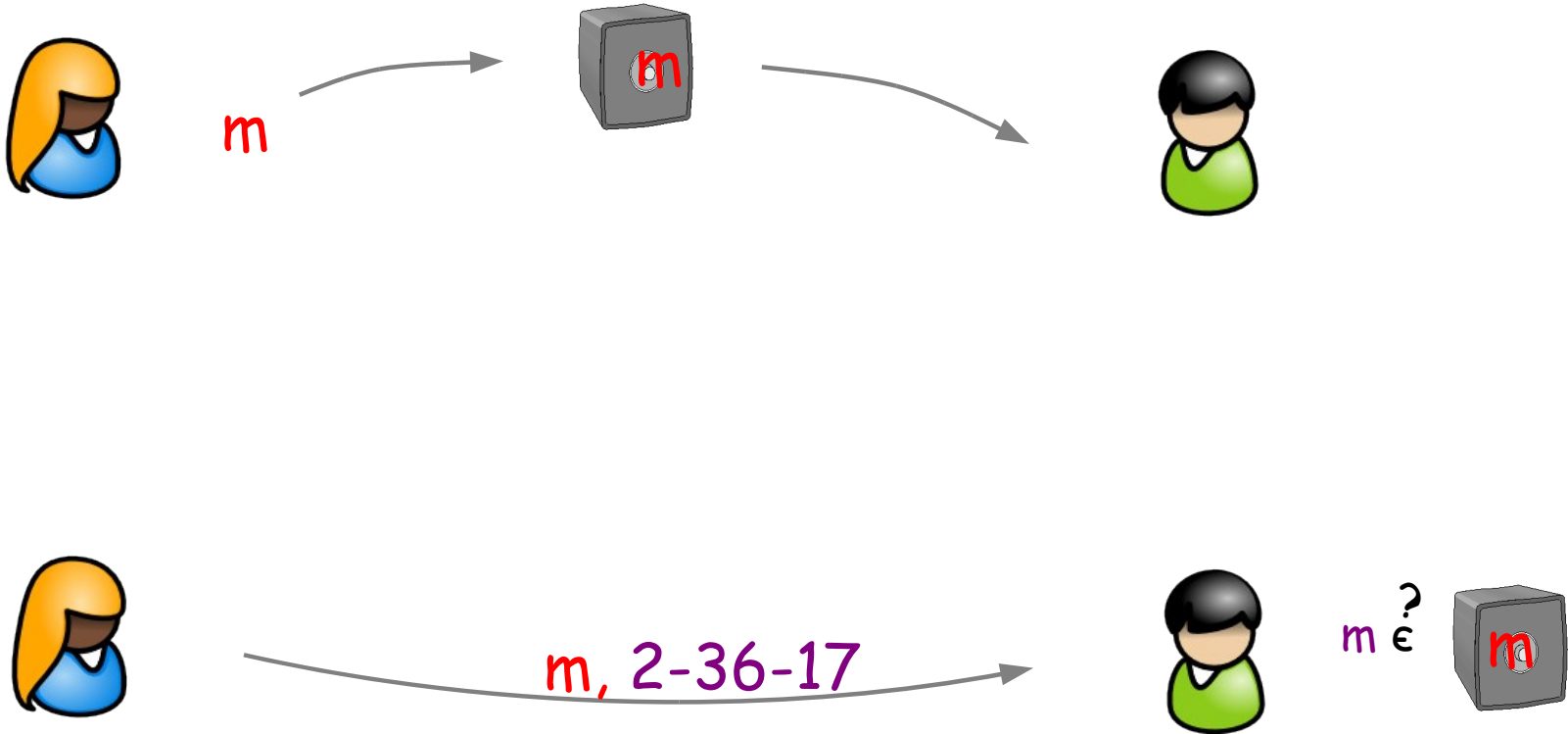
given  $g^x$  and  $g^y$  it is hard to *compute*  $g^{xy}$

## Decisional Diffie-Hellman Assumption

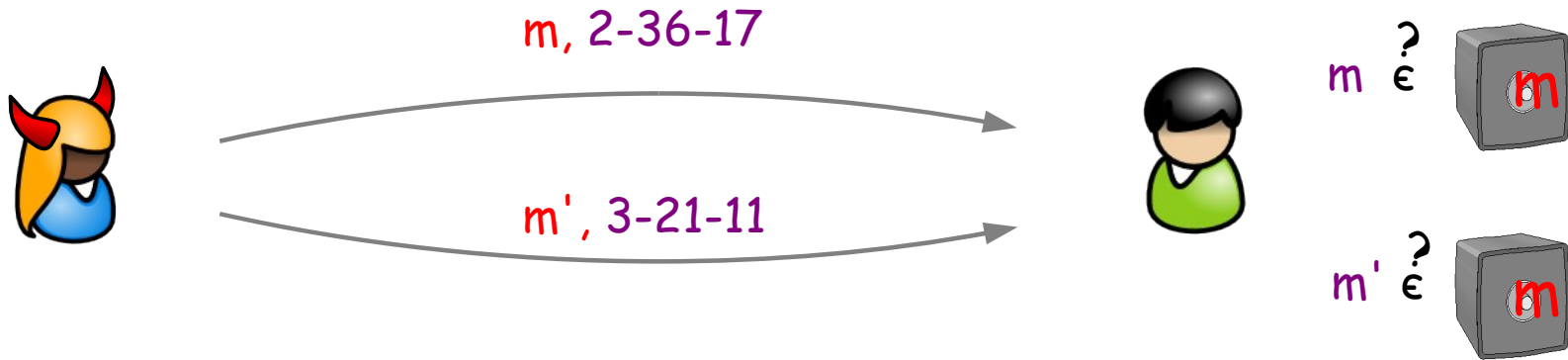
given  $g^x, g^y$ , and  $g^z$  it is hard to *decide* if  $g^z = g^{xy}$

A stack of papers or documents is lying on a textured, light-colored surface. The papers are slightly fanned out, showing their edges. The background is a mottled, light brown or tan color with a grainy texture.

**commitment scheme**



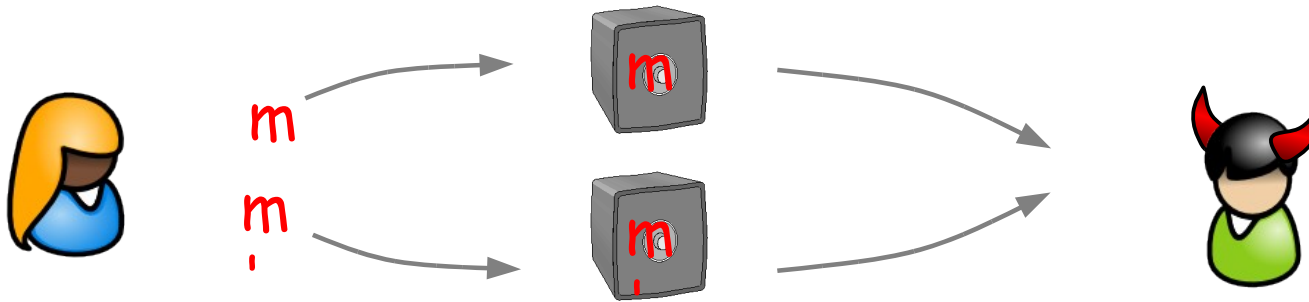
## Binding



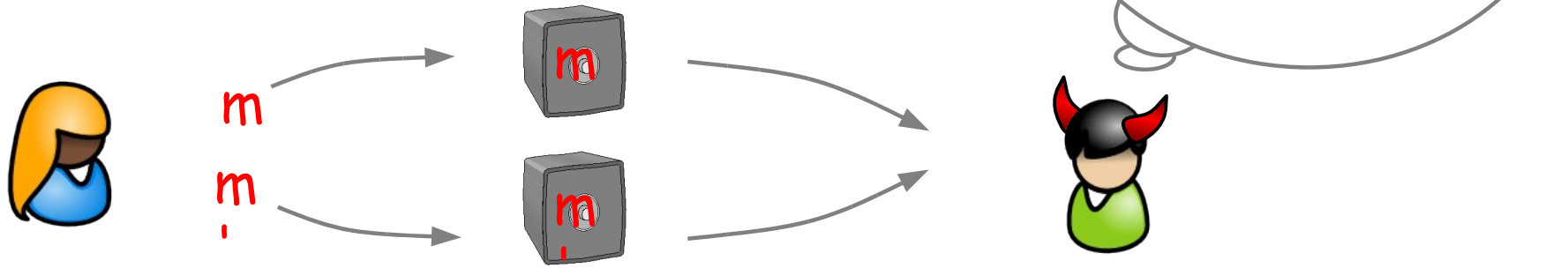
## Binding



Hiding: for all message  $m$ ,  
 $m'$



Hiding: for all message  $m$ ,  
 $m'$



Group  $G = \langle g \rangle = \langle h \rangle$  of order  $q$

To commit to element  $x \in \mathbb{Z}_q$ :

- Pedersen: perfectly hiding, computationally binding

choose  $r \in \mathbb{Z}_q$  and compute  $c = g^x h^r$

- ElGamal: computationally hiding, perfectly binding:

choose  $r \in \mathbb{Z}_q$  and compute  $c = (g^x h^r, g^r)$

To open commitment:

- reveal  $x$  and  $r$  to verifier
- verifier checks if  $c = g^x h^r$



Pedersen's Scheme:

Choose  $r \in \mathbb{Z}_q$  and compute  $c = g^x h^r$

Perfectly hiding:

Let  $c$  be a commitment and  $u = \log_g h$

$$\begin{aligned}\text{Thus } c = g^x h^r &= g^{x+ur} = g^{(x+ur') + u(r-r')} \\ &= g^{x+ur'} h^{r-r'} \quad \text{for any } r'!\end{aligned}$$

I.e., given  $c$  and  $x'$  here exist  $r'$  such that  $c = g^{x'} h^{r'}$

Computationally binding:

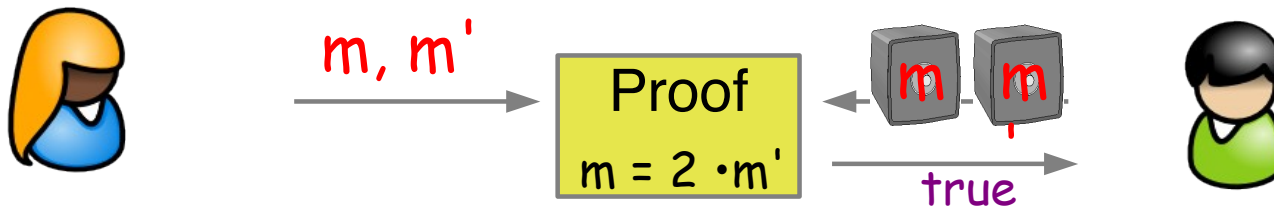
Let  $c, (x', r')$  and  $(x, r)$  s.t.  $c = g^{x'} h^{r'} = g^x h^r$

Then  $g^{x'-x} = h^{r-r'}$  and  $u = \log_g h = (x'-x)/(r-r') \pmod q$

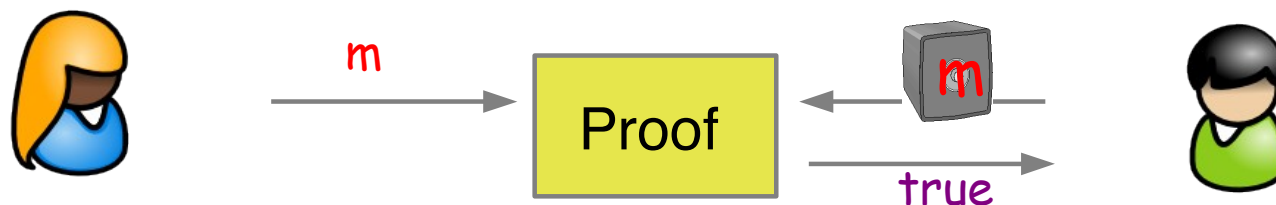
## Proof of Knowledge of Contents



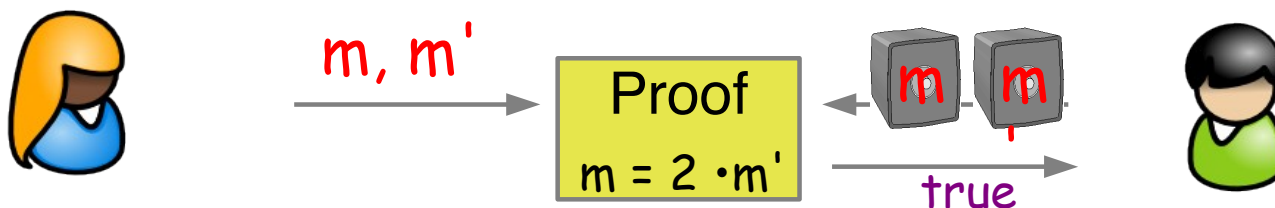
## Proof of Relations among Contents



Let  $C = g^m h^r$  and  $C' = g^{m'} h^r$  then:



PK $\{(a, \beta): C = g^\beta h^a\}$

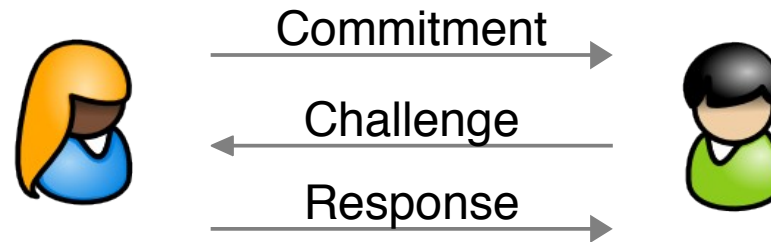


PK $\{(a, \beta, \gamma): C' = g^\beta h^a \wedge C = (g^2)^\beta h^\gamma\}$

A stack of papers or documents is lying on a textured, light-colored surface. The papers are slightly fanned out, showing their edges. The background is a mottled, light brown and beige color with a grainy texture.

zero-knowledge proofs

- interactive proof between a prover and a verifier about the prover's knowledge



- properties:

## zero-knowledge

verifier learns nothing about the prover's secret

## proof of knowledge (soundness)

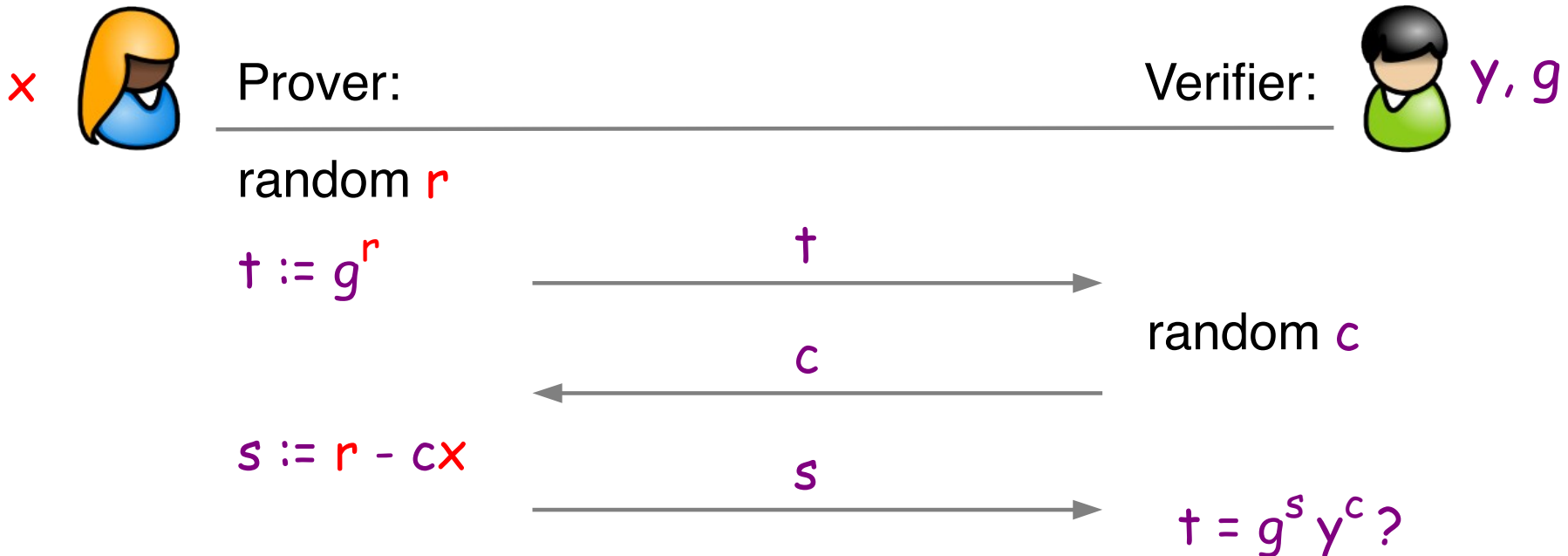
prover can convince verifier only if she knows the secret

## completeness

if prover knows the secret she can always convince the verifier

Given group  $\langle g \rangle$  and element  $y \in \langle g \rangle$ .

Prover wants to convince verifier that she *knows*  $x$  s.t.  $y = g^x$  such that verifier only learns  $y$  and  $g$ .

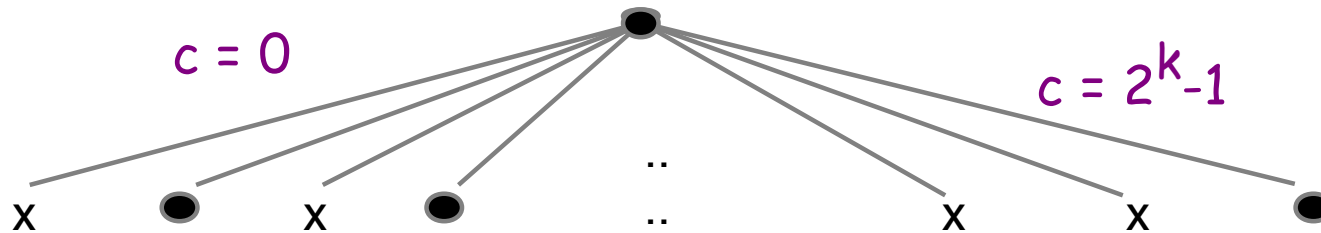


notation:  $\text{PK}\{(a): y = g^a\}$

## *Proof of Knowledge Property:*

If prover is successful with non-negl. probability, then she “knows”  $x = \log_g \gamma$ ,  
i.e., ones can extract  $x$  from her.

Assume  $c \in \{0,1\}^k$  and consider execution tree:

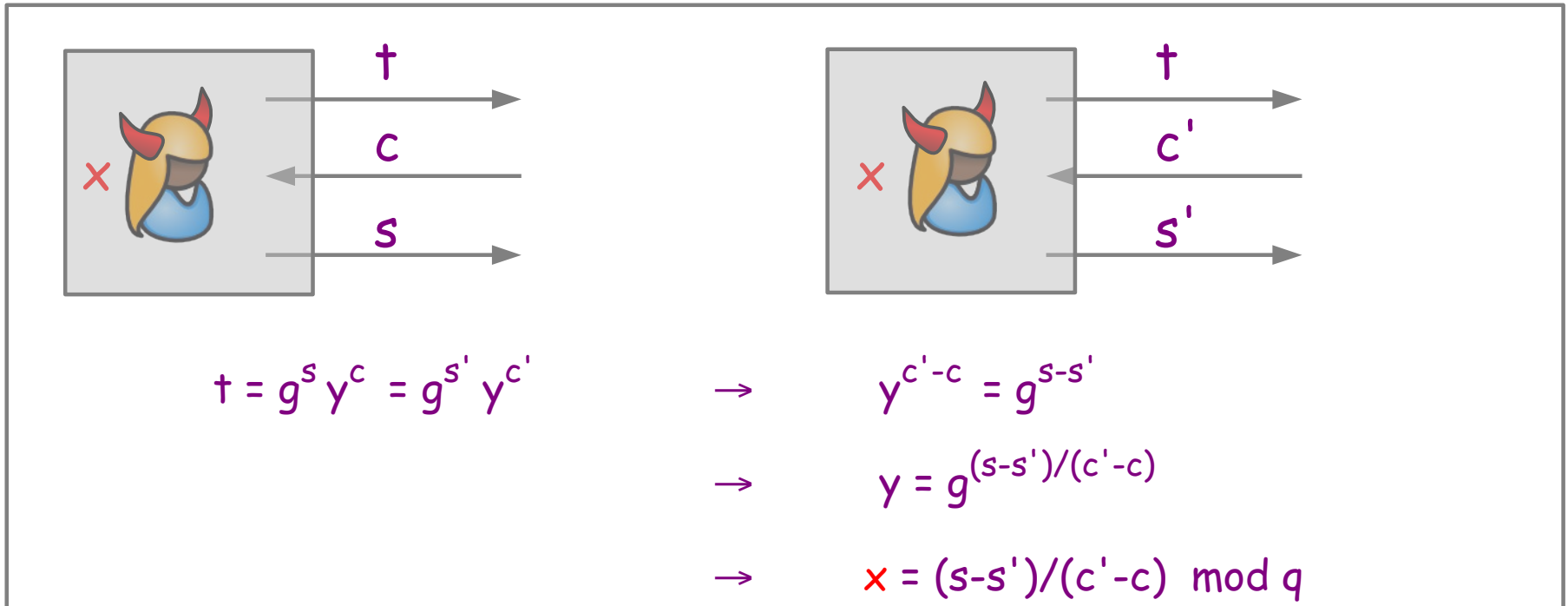


If success probability for any prover (including malicious ones)  
is  $> 2^{-k}$  then there are two *accepting* tuples  $(t, c_1, s_1)$  and  
 $(t, c_2, s_2)$  for the same  $t$ .

Prover might do protocol computation in any way it wants & we cannot analyse code.

Thought experiment:

- Assume we have prover as a black box → we can reset and rerun prover
- Need to show how secret can be extracted via protocol interface





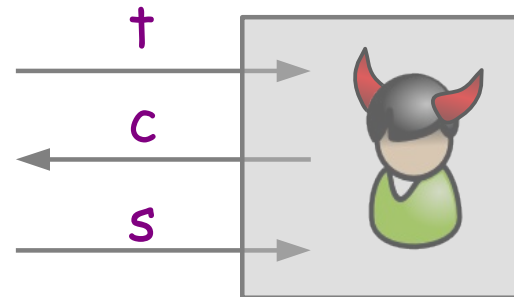
*Zero-knowledge* property:

If verifier does not learn anything (except the fact that Alice knows  $x = \log_g y$ )

Idea: One can simulate whatever Bob “sees”.

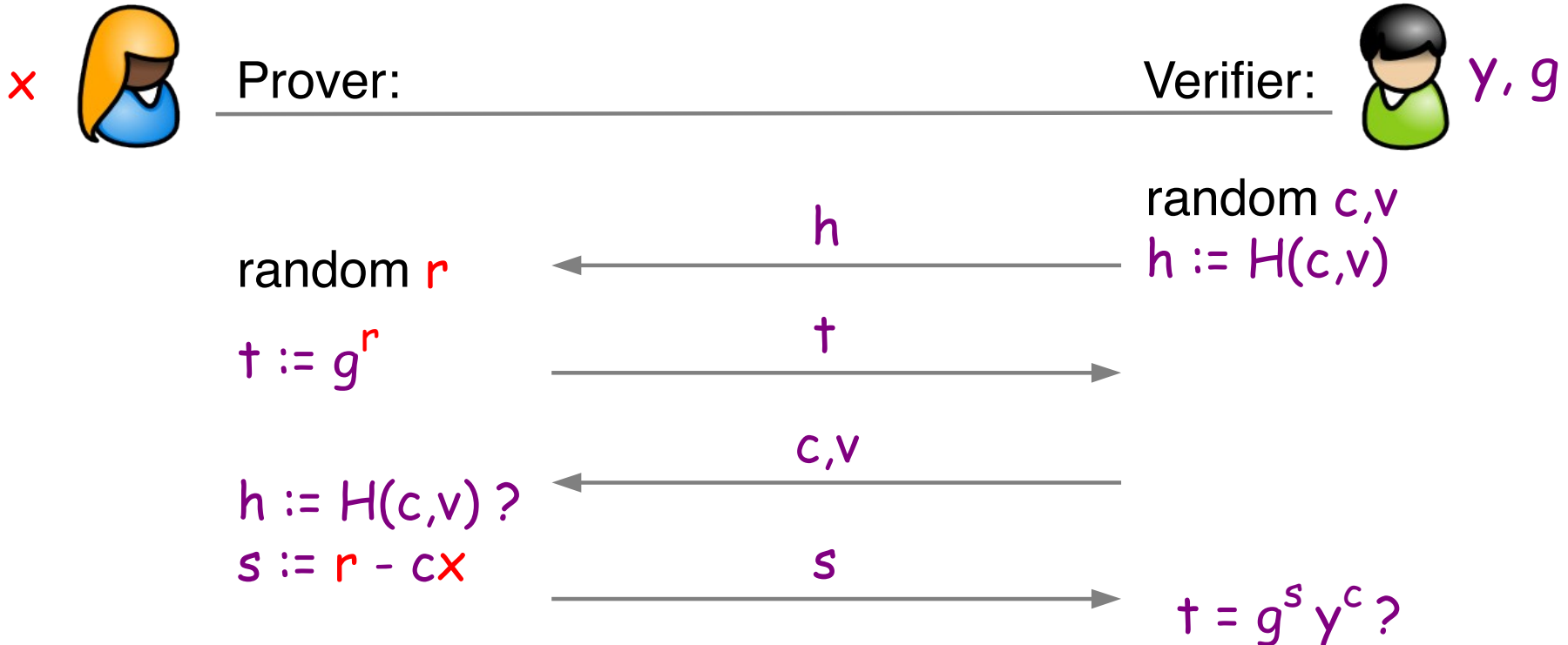
Choose random  $c', s'$   
compute  $t := g^{s'} y^{c'}$

if  $c = c'$  send  $s' = s$ ,  
otherwise restart



Problem: if domain of  $c$  too large, success probability becomes too small

One way to modify protocol to get large domain  $c$ :



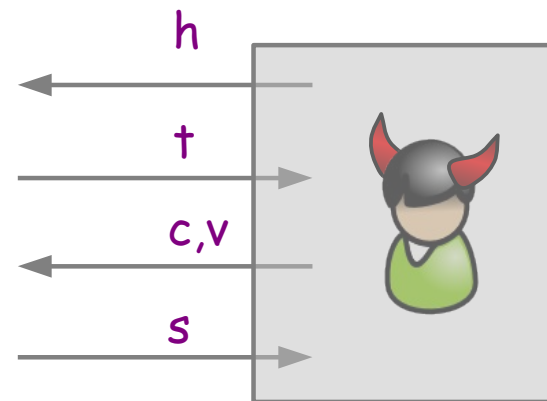
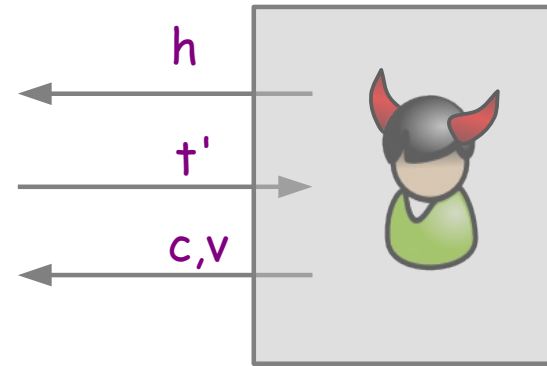
notation:  $PK\{(a): y = g^a\}$

One way to modify protocol to get large domain  $c$ :

Choose random  $c', s'$   
compute  $t' := g^{s'} y^{c'}$

after having received  $c$  “reboot”  
verifier

Choose random  $s$   
compute  $t := g^s y^c$   
send  $s$



Signature SPK $\{(a): y = g^a\}(m)$ :

Signing a message  $m$ :



- chose random  $r \in \mathbb{Z}_q$  and
- compute  $c := H(g^r || m) = H(t || m)$   
 $s := r - cx \pmod{q}$
- output  $(c, s)$

Verifying a signature  $(c, s)$  on a message  $m$ :

- check  $c = H(g^s y^c || m) ? \Leftrightarrow t = g^s y^c ?$



Security:

- underlying protocol is zero-knowledge proof of knowledge
- hash function  $H(\cdot)$  behaves as a “random oracle.”

Many Exponents:

$$\text{PK}\{(\alpha, \beta, \gamma, \delta): \gamma = g^\alpha h^\beta z^\gamma k^\delta u^\beta\}$$

Logical combinations:

$$\text{PK}\{(\alpha, \beta): \gamma = g^\alpha \wedge z = g^\beta \wedge u = g^\beta h^\alpha\}$$

$$\text{PK}\{(\alpha, \beta): \gamma = g^\alpha \vee z = g^\beta\}$$

Intervals and groups of different order (under SRSA):

$$\text{PK}\{(\alpha): \gamma = g^\alpha \wedge \alpha \in [A, B]\}$$

$$\text{PK}\{(\alpha): \gamma = g^\alpha \wedge z = g^\alpha \wedge \alpha \in [0, \min\{\text{ord}(g), \text{ord}(g)\}]\}$$

Non-interactive (Fiat-Shamir heuristic, Schnorr Signatures):

$$\text{PK}\{(\alpha): \gamma = g^\alpha\}(m)$$

Let  $g, h, C1, C2, C3$  be group elements.

Now, what does

$$\text{PK}\{(a1, \beta1, a2, \beta2, a3, \beta3): C1 = g^{a1} h^{\beta1} \wedge C2 = g^{a2} h^{\beta2} \wedge C3 = g^{a3} h^{\beta3} \wedge C3 = g^{a1} g^{a2} h^{\beta3}\}$$

mean?

→ Prover knows values  $a1, \beta1, a2, \beta2, \beta3$  such that

$$C1 = g^{a1} h^{\beta1}, \quad C2 = g^{a2} h^{\beta2} \quad \text{and}$$

$$C3 = g^{a1} g^{a2} h^{\beta3} = g^{a1 + a2} h^{\beta3} = g^{a3} h^{\beta3}$$

$$a3 = a1 + a2 \pmod{q}$$

And what about:

$$\text{PK}\{(a1, \dots, \beta3): C1 = g^{a1} h^{\beta1} \wedge C2 = g^{a2} h^{\beta2} \wedge C3 = g^{a3} h^{\beta3} \wedge C3 = g^{a1} (g^5)^{a2} h^{\beta3}\}$$

$$\rightarrow C3 = g^{a1} g^{5a2} h^{\beta3} = g^{a1 + 5a2} h^{\beta3}$$

$$a3 = a1 + 5a2 \pmod{q}$$

Let  $g, h, C1, C2, C3$  be group elements.

Now, what does

$$\text{PK}\{(\alpha_1, \dots, \beta_3): C1 = g^{\alpha_1} h^{\beta_1} \wedge C2 = g^{\alpha_2} h^{\beta_2} \wedge C3 = g^{\alpha_3} h^{\beta_3} \wedge C3 = C2^{\alpha_1} h^{\beta_3}\} \text{ mean?}$$

→ Prover knows values  $\alpha_1, \beta_1, \alpha_2, \beta_2, \beta_3$  such that

$$C1 = g^{\alpha_1} h^{\beta_1}, \quad C2 = g^{\alpha_2} h^{\beta_2} \quad \text{and}$$

$$C3 = C2^{\alpha_1} h^{\beta_3} = (g^{\alpha_2} h^{\beta_2})^{\alpha_1} h^{\beta_3} = g^{\alpha_2 \cdot \alpha_1} h^{\beta_3 + \beta_2 \cdot \alpha_1}$$

$$C3 = g^{\alpha_2 \cdot \alpha_1} h^{\beta_3 + \beta_2 \cdot \alpha_1} = g^{\alpha_3} h^{\beta_3}$$

$$\alpha_3 = \alpha_1 \cdot \alpha_2 \pmod{q}$$

And what about

$$\text{PK}\{(\alpha_1, \beta_1, \beta_2): C1 = g^{\alpha_1} h^{\beta_1} \wedge C2 = g^{\alpha_2} h^{\beta_2} \wedge C2 = C1^{\alpha_1} h^{\beta_2}\}$$

$$\rightarrow \alpha_2 = \alpha_1^2 \pmod{q}$$

Let  $g, h, C1, C2, C3$  be group elements.

Now, what does

$\text{PK}\{(\alpha_1, \dots, \beta_2): C1 = g^{\alpha_1} h^{\beta_1} \wedge C2 = g^{\alpha_2} h^{\beta_2} \wedge g = (C2/C1)^{\alpha_1} h^{\beta_2}\}$  mean?

→ Prover knows values  $\alpha, \beta_1, \beta_2$  such that

$$C1 = g^{\alpha_1} h^{\beta_1}$$

$$g = (C2/C1)^{\alpha_1} h^{\beta_2} = (C2 g^{-\alpha_1} h^{-\beta_1})^{\alpha_1} h^{\beta_2}$$

→  $g^{1/\alpha_1} = C2 g^{-\alpha_1} h^{-\beta_1} h^{\beta_2/\alpha_1}$

$$C2 = g^{\alpha_1} h^{\beta_1} h^{-\beta_2/\alpha_1} g^{1/\alpha_1} = g^{\alpha_1 + 1/\alpha_1} h^{\beta_1 - \beta_2/\alpha_1}$$

$$C2 = g^{\alpha_2} h^{\beta_2}$$

$$\alpha_2 = \alpha_1 + \alpha_1^{-1} \pmod{q}$$



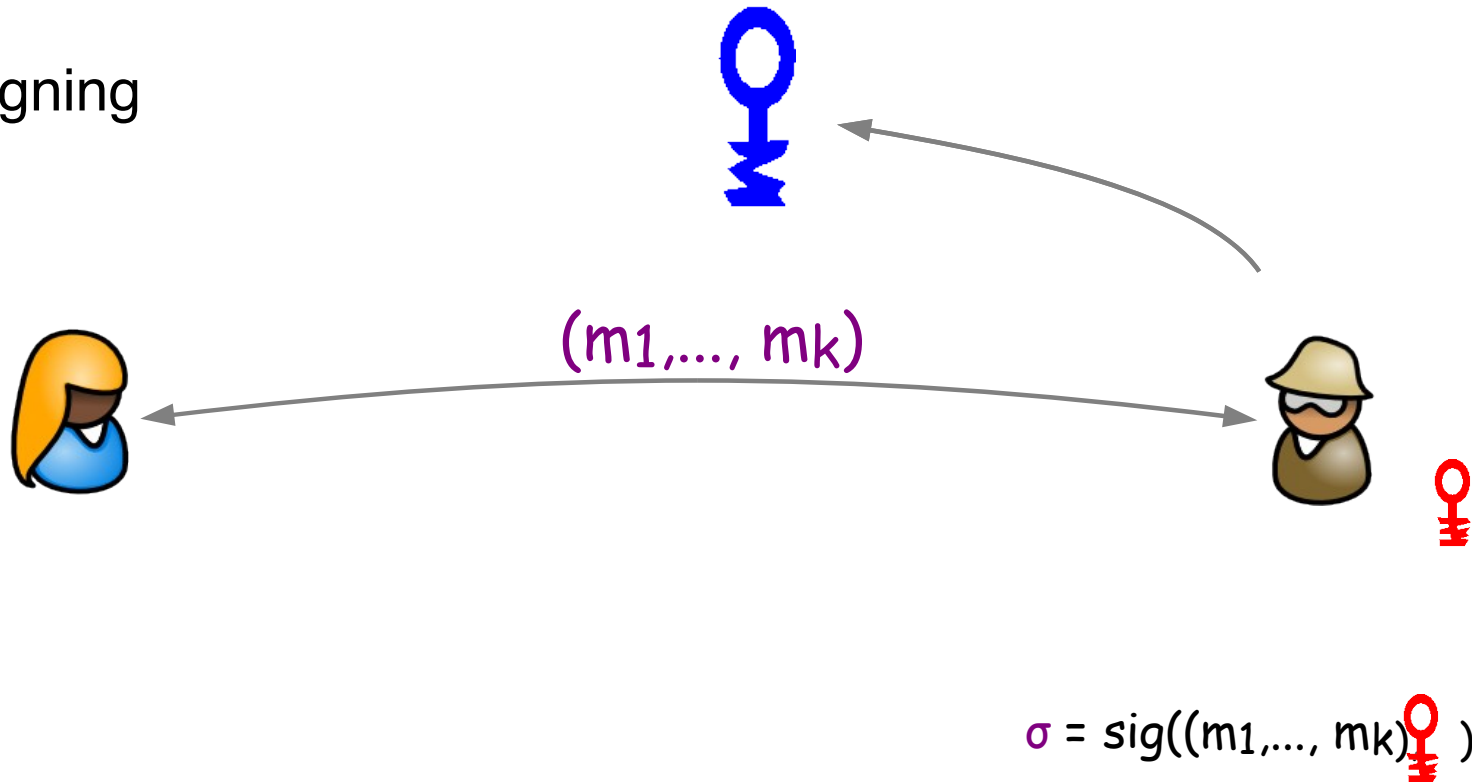


signature schemes

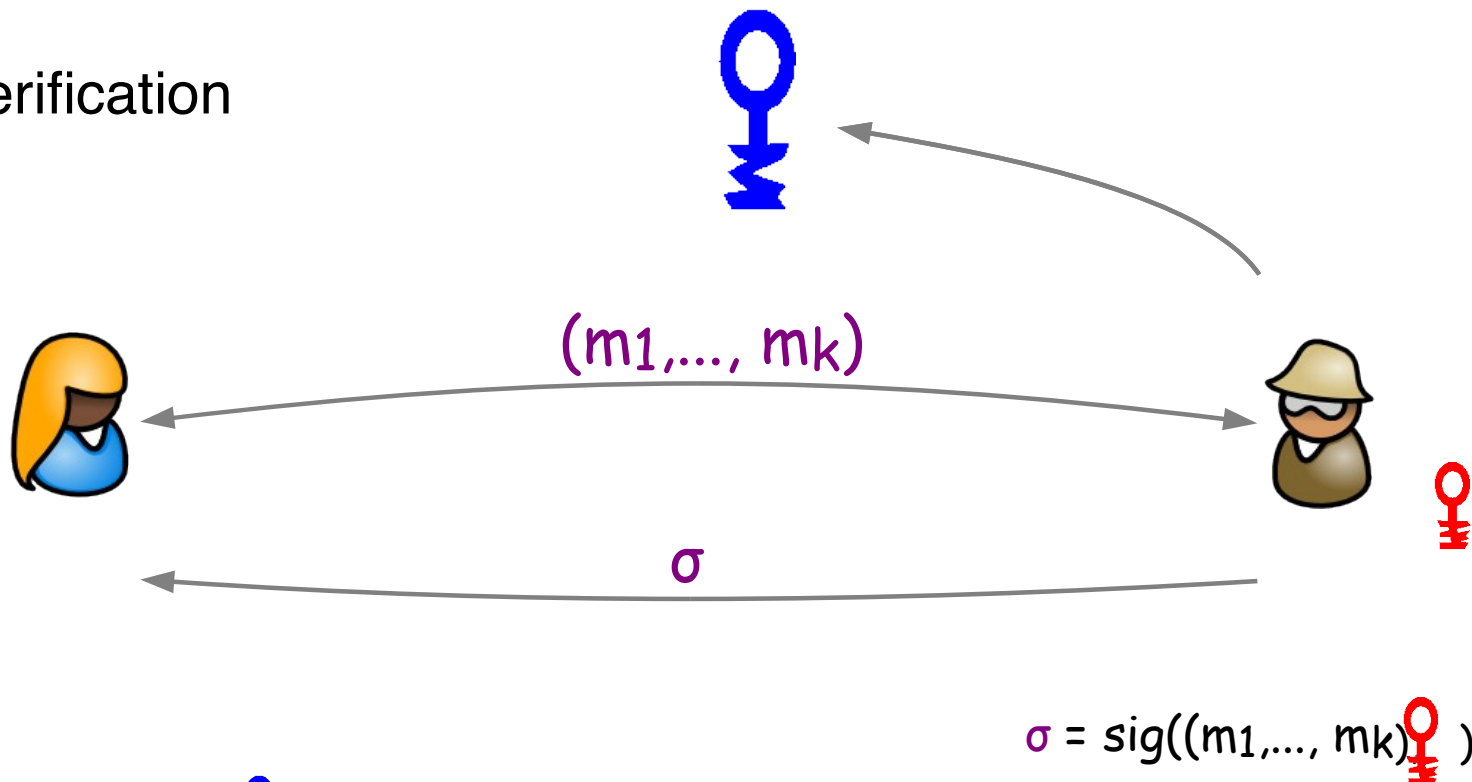
Key Generation



Signing



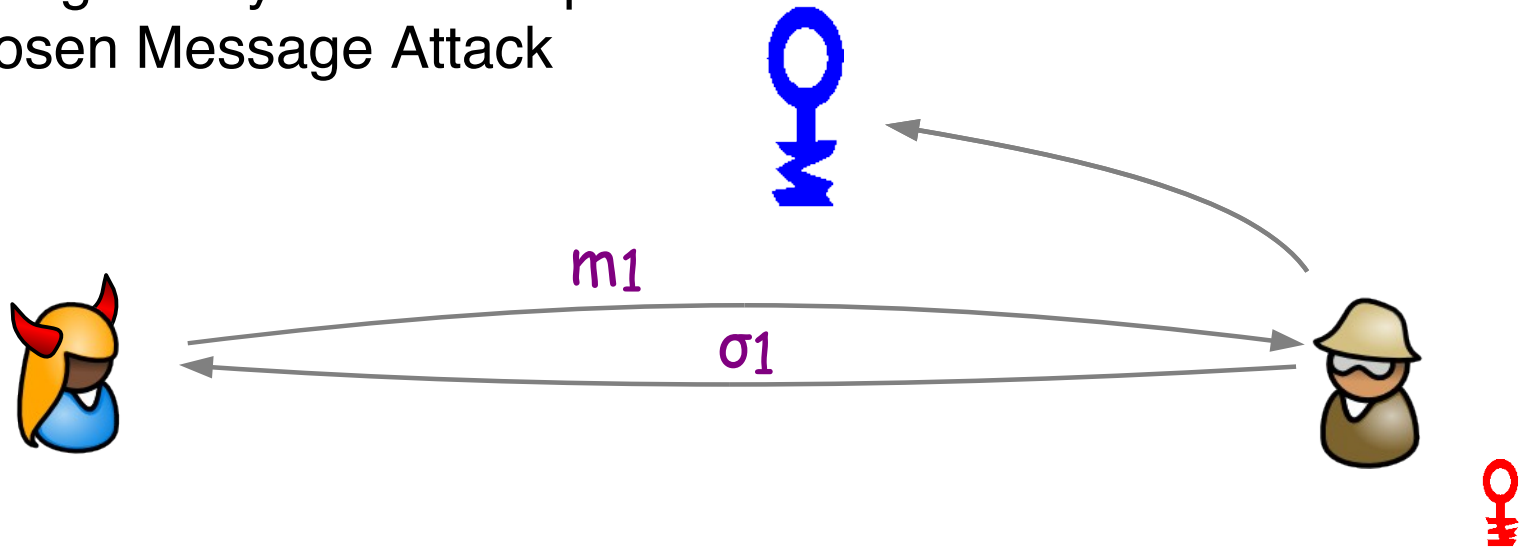
Verification



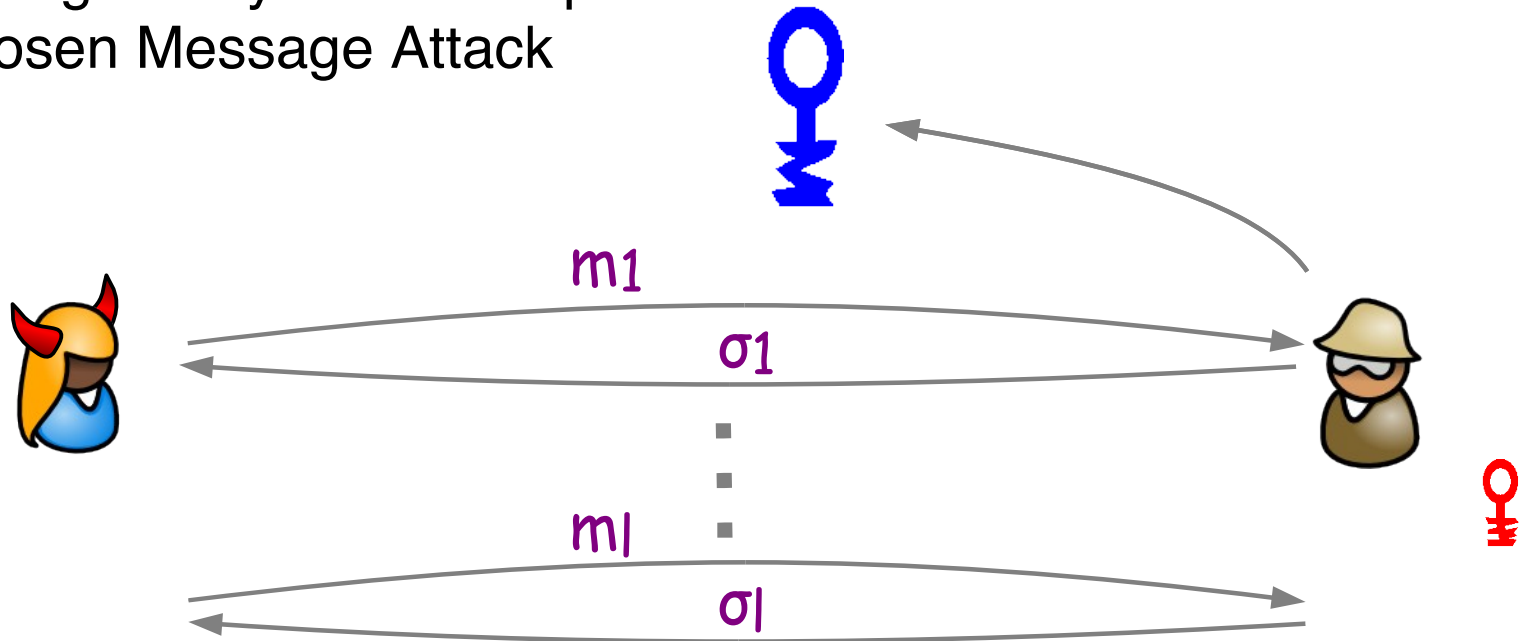
$$\text{ver}(\sigma, (m_1, \dots, m_k), \text{key}) = \text{true}$$

$$\sigma = \text{sig}((m_1, \dots, m_k), \text{key})$$

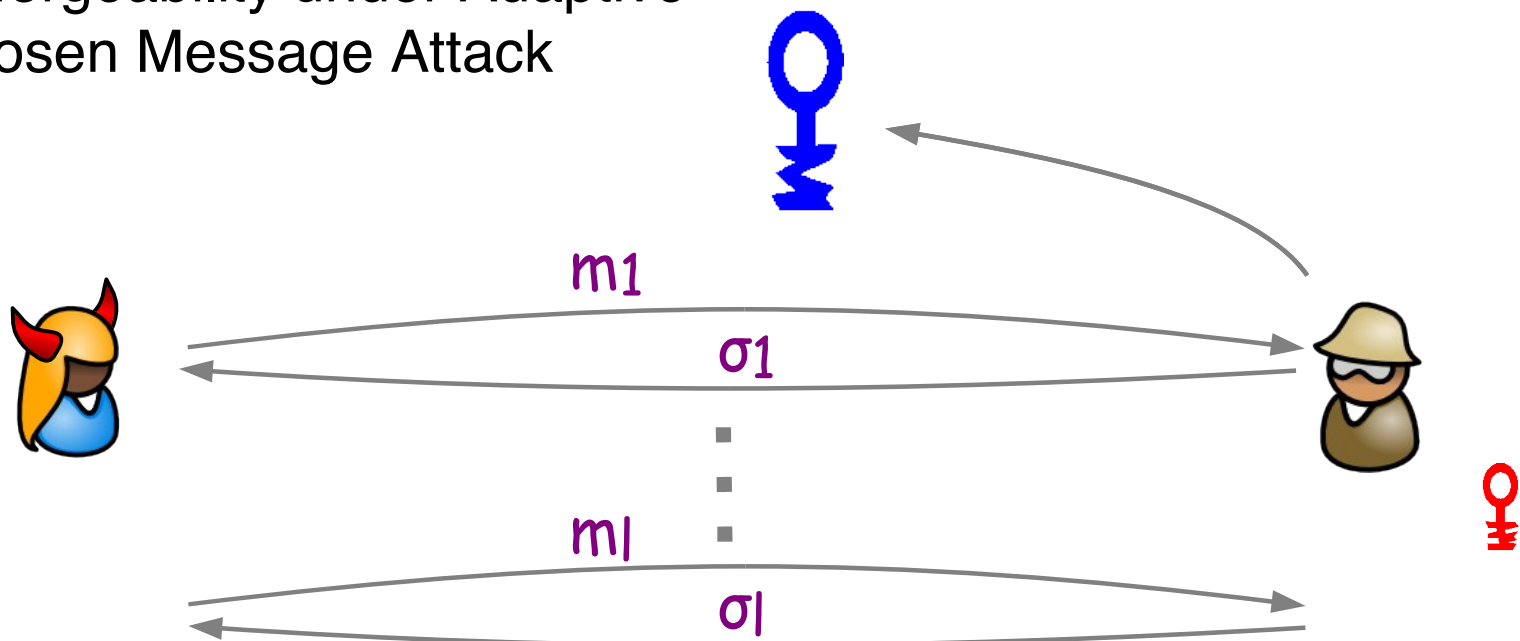
## Unforgeability under Adaptive Chosen Message Attack



## Unforgeability under Adaptive Chosen Message Attack

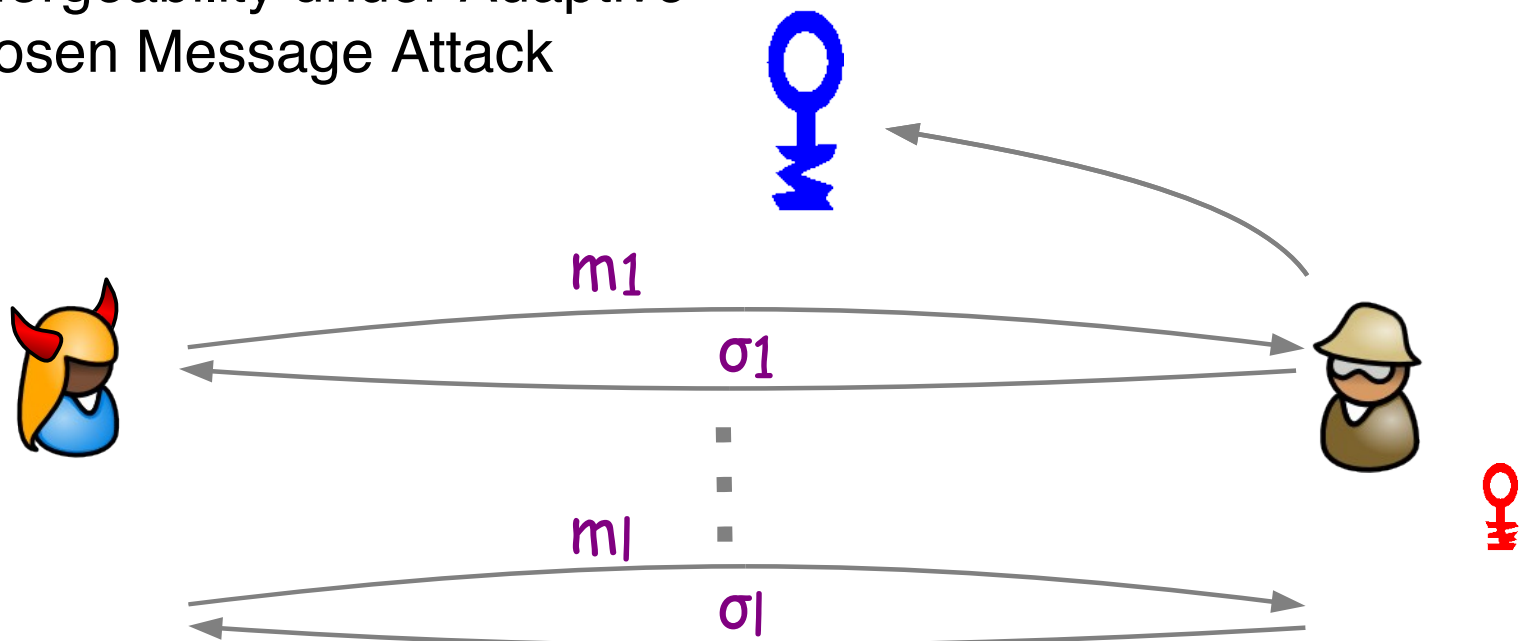


## Unforgeability under Adaptive Chosen Message Attack



$\sigma'$  and  $m' \neq m_i$  s.t.  
 $\text{ver}(\sigma', m', \text{key}) = \text{true}$

## Unforgeability under Adaptive Chosen Message Attack

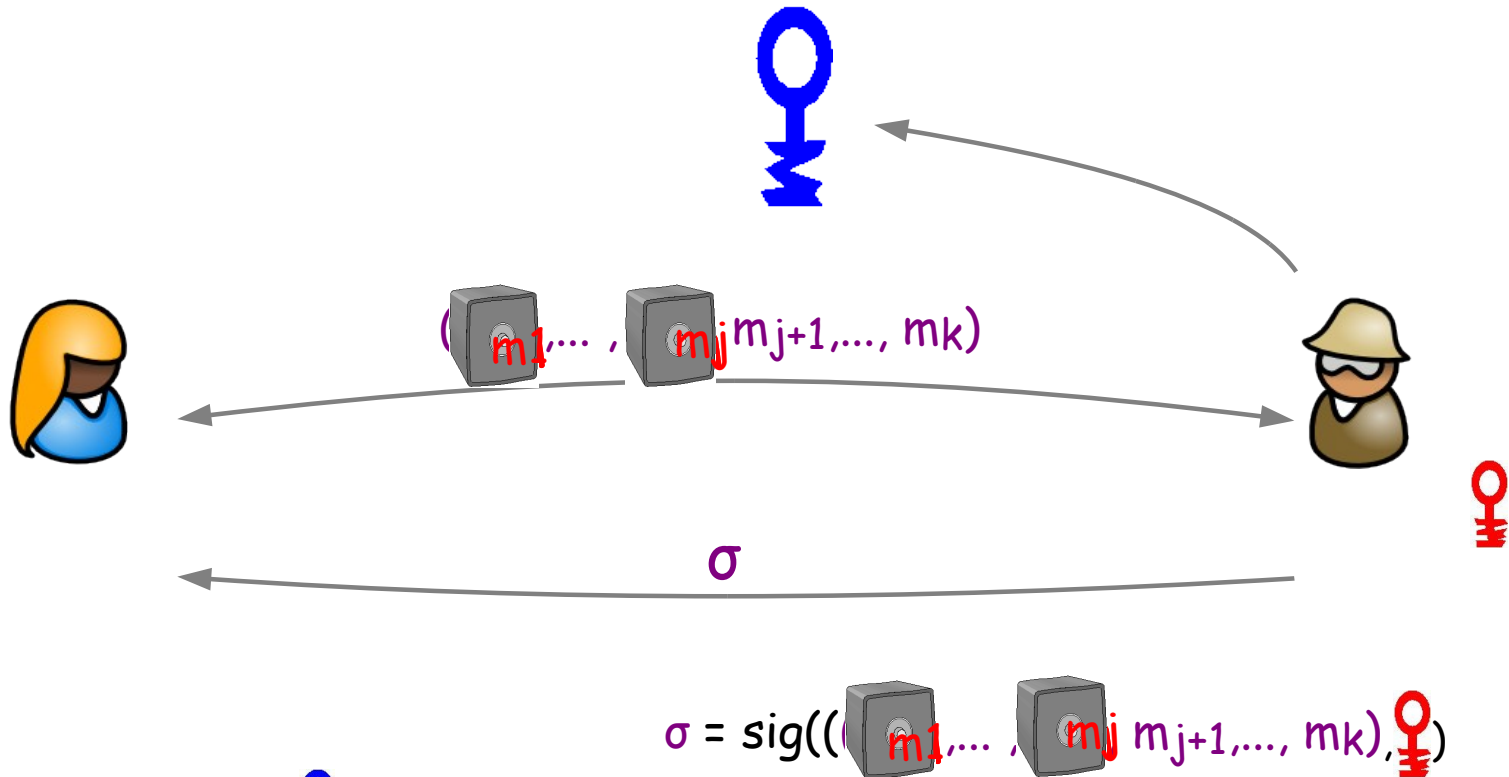


$\sigma'$  and  $m' \neq m_i$  s.t.  
 $\text{Verify}(\sigma', m', \text{pk}) = \text{true}$



A stack of papers or documents is lying on a textured, light-colored surface. The papers are slightly fanned out, and the text on them is mostly illegible. The background is a mottled, light brown and beige color.

**signature schemes with protocols**



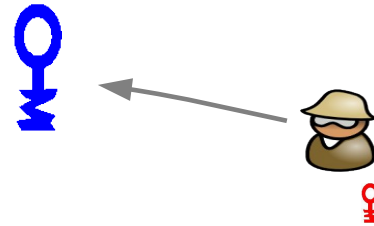
$$\text{ver}(\sigma, (m_1, \dots, m_k), \text{key}) = \text{true}$$

Verification remains unchanged!

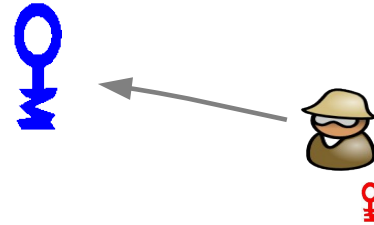
Security requirements basically the same, but

- Signer should not learn any information about  $m_1, \dots, m_j$
- Forgery w.r.t. message clear parts and opening of commitments

$\sigma$  on  $(m_1, \dots, m_k)$



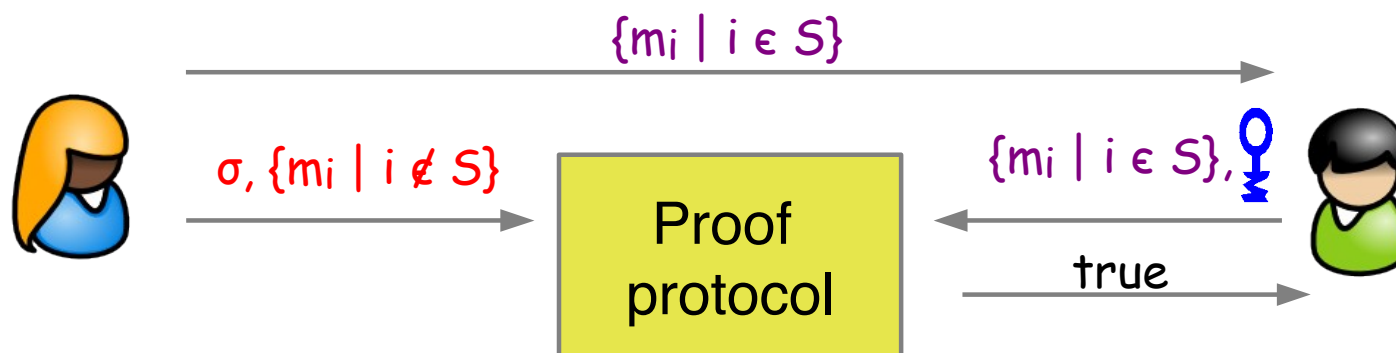
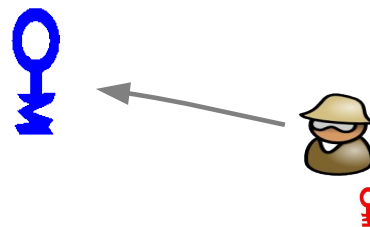
$\sigma$  on  $(m_1, \dots, m_k)$




$\{m_i \mid i \in S\}$

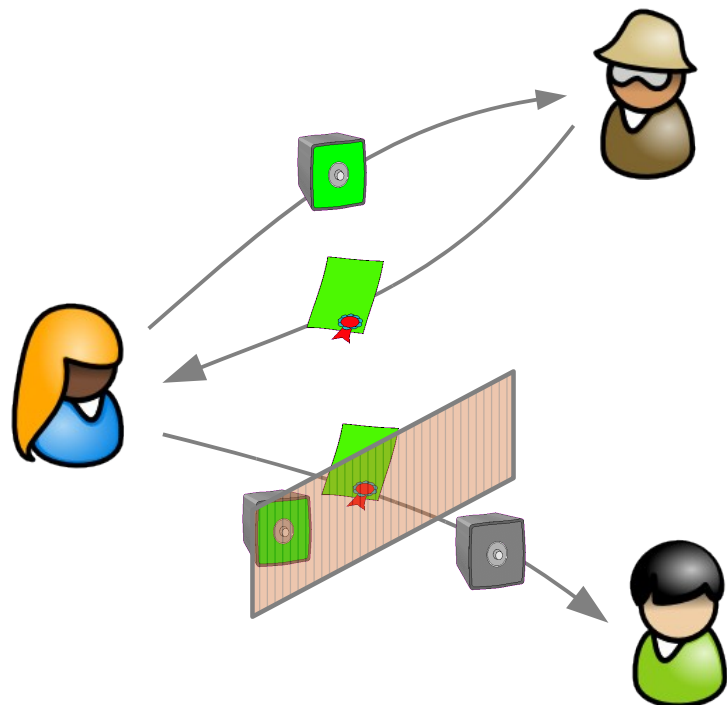


$\sigma$  on  $(m_1, \dots, m_k)$



Variation:

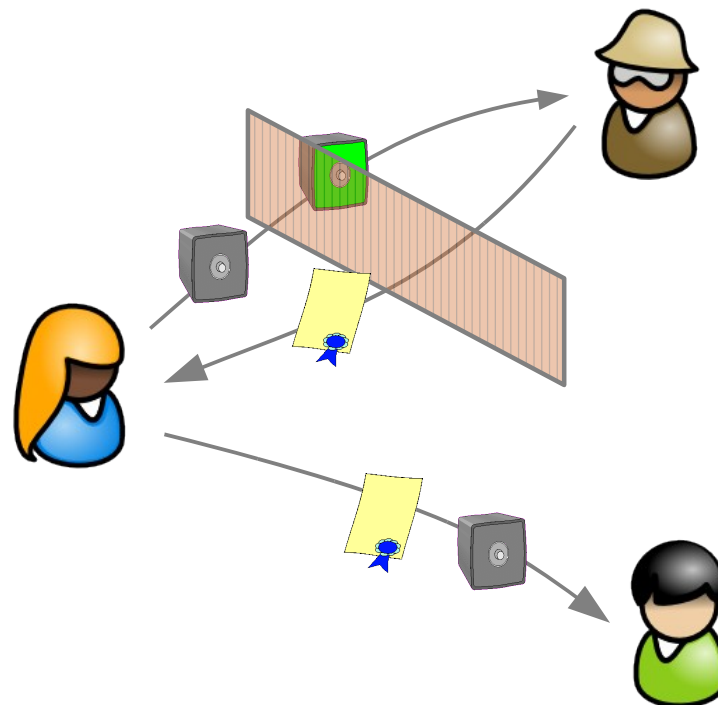
- Send also  to verifier and
- Prove that committed messages are signed
- Prove properties about hidden/committed  $m_i$



*can be used multiple times*

Damgaard, Camenisch & Lysyanskaya

Strong RSA, DL-ECC, ...



*can be used only once*

Chaum, Brands, et al.

Discrete Logs, RSA, ...

A photograph of a white SUV with a black roof rack on a gravel surface. The text "Some signature schemes" is overlaid in blue.

Some signature schemes

Rivest, Shamir, and Adleman 1978

Secret Key: two random primes  $p$  and  $q$

Public Key:  $n := pq$ , prime  $e$ ,  
and collision-free hash function

$$H: \{0,1\}^* \rightarrow \{0,1\}^l$$

Computing signature on a message  $m \in \{0,1\}^*$

$$d := 1/e \pmod{(p-1)(q-1)}$$

$$s := H(m)^d \pmod{n}$$



Verification of signature  $s$  on a message  $m \in \{0,1\}^*$

$$s^e = H(m) \pmod{n}$$

$$\text{Correctness: } s^e = (H(m)^d)^e = H(m)^{d \cdot e} = H(m) \pmod{n}$$



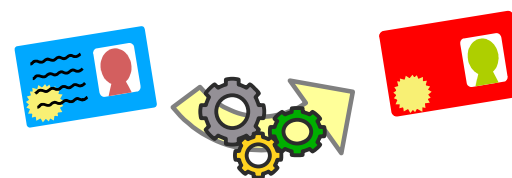
Verification signature on a message  $m \in \{0,1\}^*$

$$s^e := H(m) \pmod{n}$$




Wanna do proof of knowledge of signature on a message, e.g.,

$$\text{PK}\{ (m,s): s^e = H(m) \pmod{n} \}$$



But this is not a valid proof expression!!!! :-)

Public key of signer: RSA modulus  $n$  and  $a_i, b, d \in \mathbb{Q}\mathbb{R}_n$ , 

Secret key: factors of  $n$

To sign  $k$  messages  $m_1, \dots, m_k \in \{0,1\}^\ell$ :

- choose random *prime*  $2^{\ell+2} > e > 2^{\ell+1}$  and *integer*  $s \approx n$
- compute  $c$ :

$$c = (d / (a_1^{m_1} \cdot \dots \cdot a_k^{m_k} b^s))^{1/e} \bmod n$$

- signature is  $(c, e, s)$

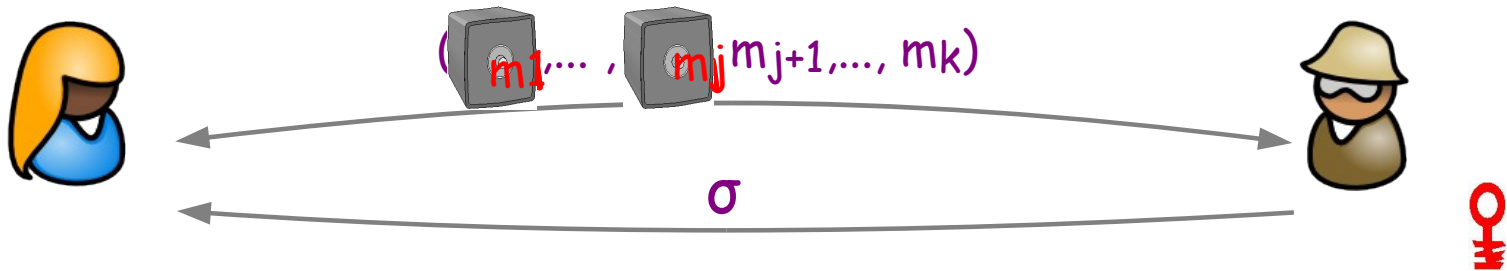


To verify a signature  $(c, e, s)$  on messages  $m_1, \dots, m_k$ :

- $m_1, \dots, m_k \in \{0,1\}^\ell$ :
- $e > 2^{\ell+1}$
- $d = c^e a_1^{m_1} \dots a_k^{m_k} b^s \pmod n$



Theorem: *Signature scheme is secure against adaptively chosen message attacks under Strong RSA assumption.*



$$\sigma = \text{sig}((m_1, \dots, m_j, m_{j+1}, \dots, m_k), \text{key})$$

$$C = a_1^{m_1} a_2^{m_2} b^{s'}$$

$$C + \text{PK}\{(m_1, m_2, s') : C = a_1^{m_1} a_2^{m_2} b^{s'}\}$$

Choose  $e, s''$

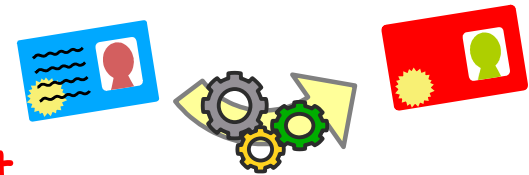
$$c = (d / (C a_3^{m_3} b^{s''}))^{1/e} \text{ mod } n$$

$$(c, e, s'')$$

$$d = c^e a_1^{m_1} a_2^{m_2} a_3^{m_3} b^{s'+s''} \text{ mod } n$$

Recall:  $d = c^e a_1^{m_1} a_2^{m_2} b^s \pmod n$

Observe:



- Let  $c' = c b^t \pmod n$  with randomly chosen  $t$
- Then  $d = c'^e a_1^{m_1} a_2^{m_2} b^{s-et} \pmod n$ , i.e.,  $(c', e, s^* = s-et)$  is also signature on  $m_1$  and  $m_2$

To prove knowledge of signature  $(c', e, s^*)$  on  $m_2$  and some  $m_1$

- provide  $c'$
- $\text{PK}\{(\epsilon, \mu_1, \sigma) : d/a_2^{m_2} := c'^\epsilon a_1^{\mu_1} b^\sigma \wedge \mu \in \{0,1\}^\ell \wedge \epsilon > 2^{\ell+1}\}$

→ proves  $d := c'^\epsilon a_1^{\mu_1} a_2^{m_2} b^\sigma$

A stack of US dollar bills is shown from a high-angle perspective, resting on a dark, textured surface. The bills are slightly fanned out, and the lighting creates a strong shadow to the right of the stack. The text 'Realizing On-Line eCash' is overlaid in blue on the left side of the image.

# Realizing On-Line eCash



choose random  $\#, s'$   
and compute

$$C = a_1 \# b^{s'}$$



$C + \text{proof}$

$(c, e, s'')$



Choose  $e, s''$

$$c = (d / (C b^{s''}))^{1/e} \pmod n$$



$(c, e, s'' + s')$  s.t.

$$d = c^e a_1 \# b^{s'' + s'} \pmod n$$



$(c, e, s'' + s')$  s.t.

$$d = c^e a_1^\# b^{s'' + s'} \pmod{n}$$



$\#, c', \text{proof}$



compute

$$c' = c b^{s'} \pmod{n}$$

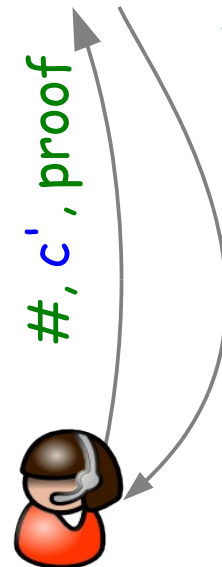
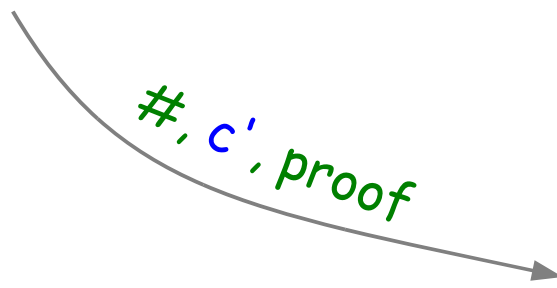
$$\text{proof} = \text{PK}\{(\varepsilon, \mu, \rho, \sigma) : d / a_1^\# = c'^\varepsilon b^\sigma \pmod{n}\}$$

$$(c, e, s'' + s') \text{ s.t.}$$

$$d = c^e a_1^\# b^{s'' + s'} \pmod{n}$$



#  $\in L$ ?



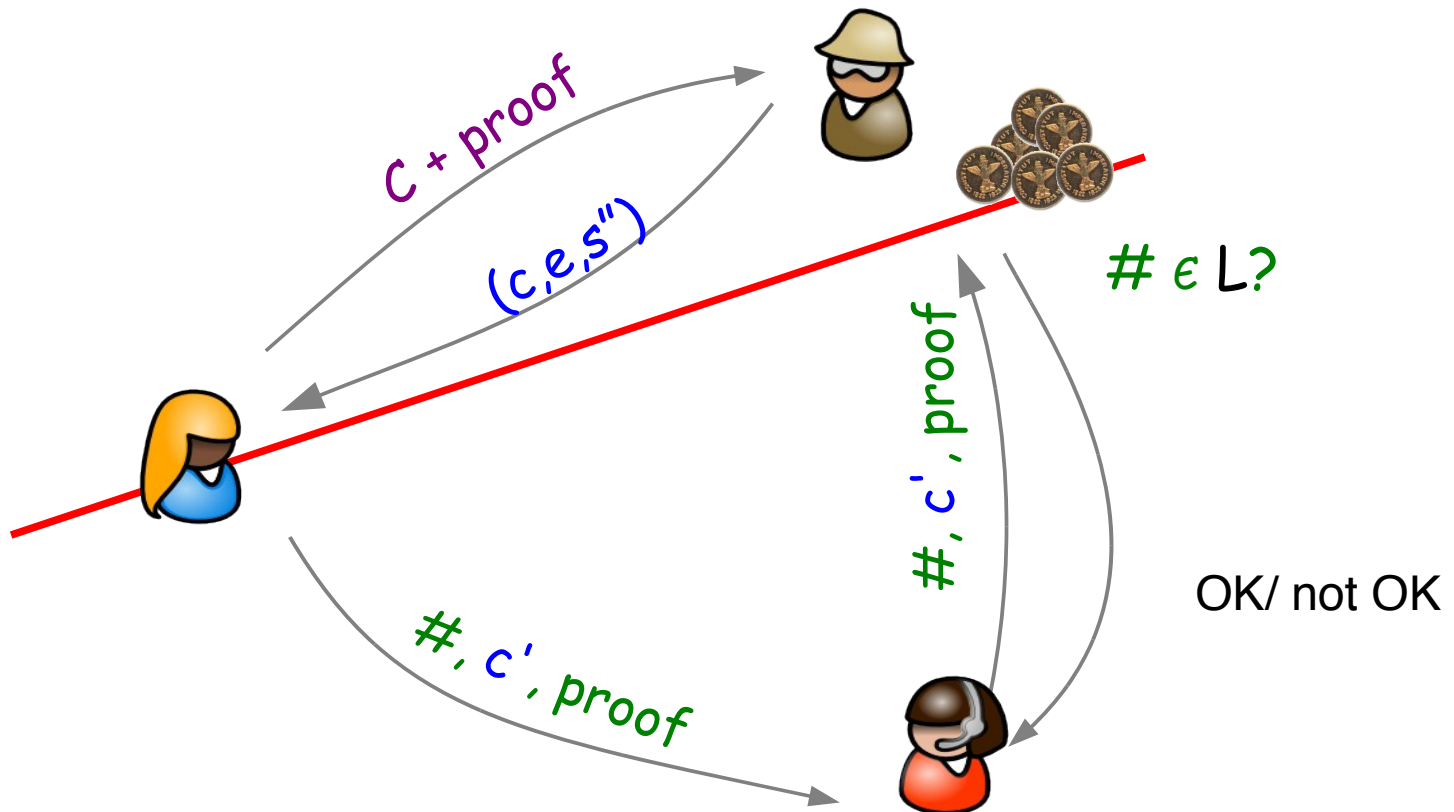
OK/ not OK

compute

$$c' = c b^{s'} \pmod{n}$$

$$\text{proof} = \text{PK}\{(\epsilon, \mu, \rho, \sigma) : d / a_1^\# = c'^\epsilon b^\sigma \pmod{n}\}$$

- Anonymity
  - Bank does not learn # during withdrawal
  - Bank & Shop do not learn  $c$ ,  $e$  when spending



## Double Spending:

- Spending = Compute


- $c' = c b^{s'} \pmod n$

- $\text{proof} = \text{PK}\{(\varepsilon, \mu, \rho, \sigma) : d / a_1^{\#} = c'^{\varepsilon} b^{\sigma} \pmod n\}$

- Can use the same # only once....

  - If more #'s are presented than withdrawals:

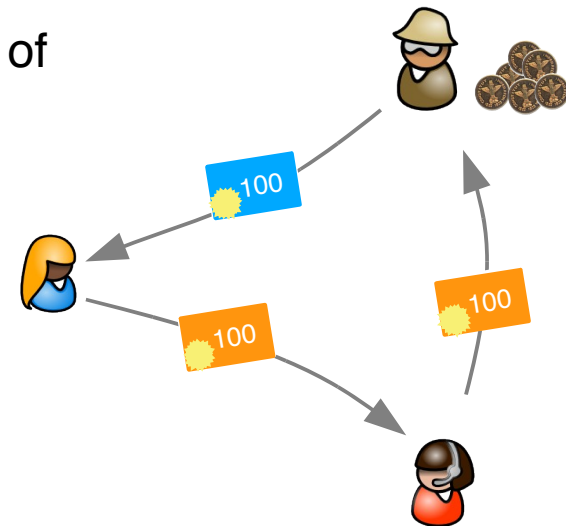
    - Proofs would not sound
    - Signature scheme would not secure

A stack of US dollar bills is shown on a textured, light-colored surface. The bills are slightly fanned out, and the top bill is clearly visible. The background is a mottled, light brown and beige color with a grainy texture.

# Realizing Off-Line eCash

## On-Line Solution:

1. Coin = random serial # (chosen by user) signed by Bank
2. Bank signs blindly
3. Spending by sending # and prove knowledge of signature to Merchant
4. Merchant checks validity w/ Bank
5. Bank accepts each serial # only once.

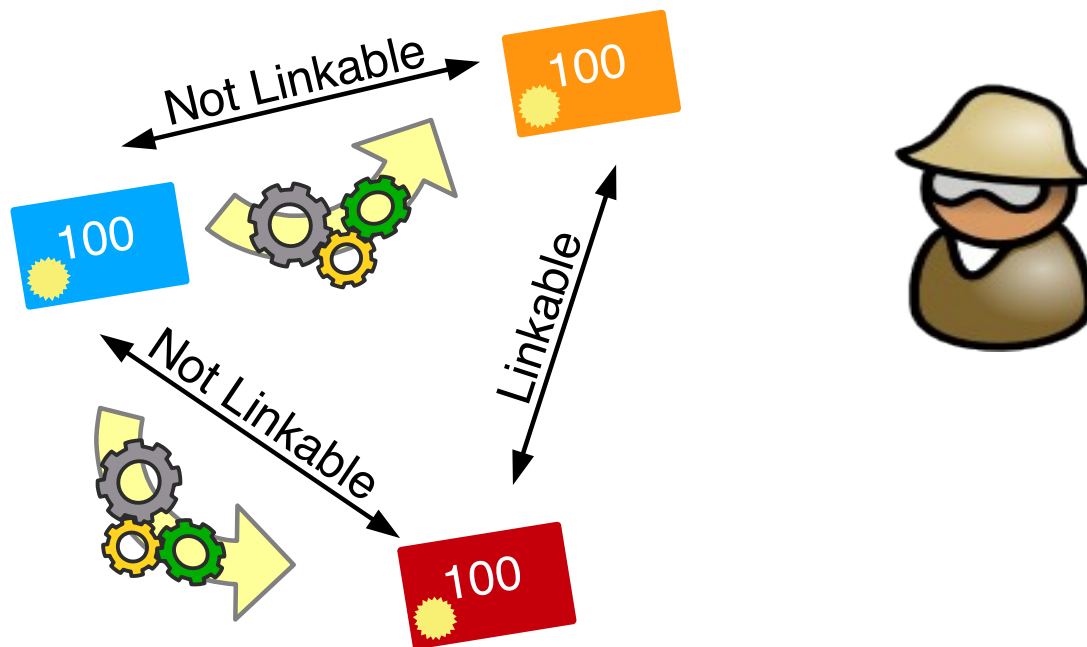


## Off-Line:

- Can check serial # only after the fact
- ... but at that point user will have been disappeared...

## Goal:

- spending coin once: OK
- spending coin twice: anonymity revoked



Seems like a paradox, but crypto is all about solving paradoxical problems :-)

## Main Idea:

- Include  $\#$ ,  $id$ ,  $r$
- Upon spending:
  - reveal  $\#$ , and  $t = id + r u$ ,
  - with  $c$  randomly chosen by merchant
- $t$  won't reveal anything about  $id$ !
- However, given two equations (for the same  $\#$ ,  $id$ ,  $r$ )
  - $t1 = id + r u1$
  - $t2 = id + r u2$one can solve for  $id$ .



choose random  $\#, r, s'$

and compute

$$C = a_1^\# a_2^r b^{s'}$$



$C + \text{proof}$

$(c, e, s'')$



$$d = c^e C a_3^{\text{nym}} b^{s''} \pmod n$$

$(c, e, s'' + s')$  s.t.

$$d = c^e a_1^\# a_2^r a_3^{\text{nym}} b^{s'' + s'} \pmod n$$



Let  $G = \langle g \rangle$  be a group of order  $q$

$(c, e, s'' + s')$  s.t.

$$d = c^e a_1^\# a_2^r a_3^{nym} b^{s'' + s'} \pmod{n}$$



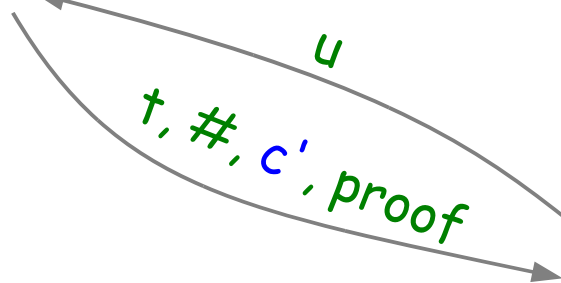
compute

$$t = r + u \text{nym} \pmod{q}$$

$$c' = c b^{s'} \pmod{n}$$

proof = PK $\{(\epsilon, \mu, \rho, \sigma) :$

$$d / a_1^\# = c'^\epsilon a_2^\rho a_3^\mu b^\sigma \pmod{n} \wedge g^t = g^\rho (g^u)^\mu \}$$



choose random  $u$



PK $\{(\varepsilon, \mu, \rho, \sigma) :$

$$d / a_1^{\#} = c'^{\varepsilon} a_2^{\rho} a_3^{\mu} b^{\sigma} \pmod{n} \wedge g^{\dagger} = g^{\rho} (g^{\mu})^{\mu} \}$$

$$1. \quad d = c'^{\varepsilon} a_1^{\#} a_2^{\rho} a_3^{\mu} b^{\sigma} \pmod{n}$$

$\Rightarrow (c', \varepsilon, \sigma)$  is a signature on  $(\#, \mu, \rho)$

$$2. \quad g^{\dagger} = g^{\rho + \mu \mu}$$

$$\Rightarrow \dagger = \rho + \mu \mu \pmod{q},$$

i.e.,  $\dagger$  was computed correctly!

#  $\in L$ ?

- If so:
1.  $t = \rho + u \mu \pmod{q}$
  2.  $t' = \rho + u' \mu \pmod{q}$

solve for  $\rho$  and  $\mu$ .

$\Rightarrow \mu = nym$  because of proof



$u, t, \#,$   
proof

- Unforgeable:
  - no more coins than  $\#$ 's,
    - otherwise one can forge signatures
    - or proofs are not sound
  - if coins with same  $\#$  appears with different  $u$ 's  $\Rightarrow$  reveals  $nym$
  
- Anonymity:
  - $\#$  and  $r$  are hidden from signer upon withdrawal
  - $t$  does not reveal anything about  $nym$  (is blinded by  $r$ )
  - proof  $proof$  does not reveal anything

## e-Cash

- K-spendable cash
  - Multiple serial numbers and randomizers per coin
  - Use PRF to generate serial number and randomizers from seed in coin
- Money laundering preventions
  - Must not spend more than \$10000 dollars with same party
  - Essentially use additional coin defined per merchant that controls this

Other protocols from these building blocks, essentially anything with authentication and privacy

- Anonymous credentials, eVoting, ....

## Alternative building blocks

- A number of signatures scheme that fit the same bill
- (Verifiable) encryption schemes that work along as well
- Alternative framework: Groth-Sahai proofs plus “structure-preserving” schemes

PhD and Postdocs available at IBM Research – Zurich  
Please contact me

# Thank you!

- eMail: [identity@zurich.ibm.com](mailto:identity@zurich.ibm.com)
- Links:
  - [www.abc4trust.eu](http://www.abc4trust.eu)
  - [www.futureID.eu](http://www.futureID.eu)
  - [www.au2eu.eu](http://www.au2eu.eu)
  - [www.PrimeLife.eu](http://www.PrimeLife.eu)
  - [www.zurich.ibm.com/idemix](http://www.zurich.ibm.com/idemix)
  - [idemixdemo.zurich.ibm.com](http://idemixdemo.zurich.ibm.com)
- Code
  - [github.com/p2abcengine](https://github.com/p2abcengine) & [abc4trust.eu/idemix](http://abc4trust.eu/idemix)



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